The Group Commutator Length in Terms of Group Ring

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Let G be a group and [G, G] be its commutator subgroup which is generated by special elements named commutators. Therefore, every element from the commutator subgroup can be represented as a product of commutators. Such representation of an element g from [G, G] that contains the least possible number of commutators is called a minimal commutator representation of g. The number of commutators in a minimal representation of g is called the commutator length of g. The purpose of our work is creation of the instrument for transfer the work with commutator length from the group G to its group ring ZG. We give an equivalent definition of commutator length for element g from [G, G], which does not use concept of group commutator, but uses concept of ring commutator in the group ring. Now we introduce the following concept. Suppose x, y is from ZG; then the element xy - yx is called the ring commutator of x, y. Suppose g is from [G, G]; then there exist a representation of element g - 1 as sum of ring commutators, multiplied on elements from G. The least number of summands in the representation is called ring commutator length of element g - 1. The aim of this paper is to prove the equality commutator length of g and ring commutator length of g - 1. Therefore, ring commutator length is equal to the group commutator length. Consequently, it is another definition of the well-known object. We believe that the general commutator length theory will greatly benefit from using this definition.