Comparison of the Smoothness of an Outer Function and Its Modulus

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We study the smooth properties of the analytic functions on the unit circle. Namely, we consider the classical problem of comparing the smoothness of an analytic function and the smoothness of its modulus. Our research is reduced to the case of outer analytic functions. The problem stated above was treated in several articles. As a common phenomenon in this field of research, we stress out the following result: without any additional restrictions one can expect the two-fold decrease of the smoothness of an outer function comparing to the smoothness of its modulus. Moreover, some additional restriction allows one to reduce the value of the above-mentioned decrease or avoid it all. Thus according to the recent results, the correlation between the amount of this decrease and some properties of the logarithm of an outer function's modulus boundary values is predicted. Our goal is to give a constructive description of this correlation. We construct the exact conditions on the logarithm of an outer function's modulus boundary values that describe the case when the outer function's modulus of continuity and the modulus of continuity of the outer function's modulus are in a fixed ratio.