Embedding a Flat Torus in Three Dimensional Euclidean Space

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In the 1950s Nash and Kuiper proved the existence of an isometric embedding of a flat torus in 3D Euclidean space; however, they did not provide a visualization of such embedding. In the '70s and '80s, Gromov developed the convex integration technique, providing the tool for such construction. From 2006 to 2012, three French mathematics institutions collaborated in the Hevea Project to produce a 3D model of such embedding. My initial attempt to provide a visualization of the embedded flat torus was inspired mainly by the convex integration technique: wrapping a high-frequency but small-amplitude sine wave around a unit circle. In order to achieve the convergence to the unit circle and the desired arc length, the amplitude was decreased accordingly as the frequency increased. Unfortunately, the first derivative did not converge as the frequency approached infinity. Consequently, a new construction, inspired by the self-similarity and fractal structure of the Hevea Project solution, was created to correct the initial shortcomings. In this new technique, which was coined the sinusoidal fractal, increasingly higher frequency sine waves were wrinkled normal to the previous iteration, instead of corrugated along a single curve. I further proved that this construction satisfies all four conditions for an isometric embedding from 1D to 2D. Utilizing sinusoidal fractal curves, a model of the isometric embedding of a flat torus in 3D was constructed and proved to satisfy the criteria of isometric embeddings, and a physical realization using a 3D printer was also produced.

Awards Won:

American Mathematical Society: Award to participate in summer school "Web Valley" in Trento, Italy