## The Investigation of an Impartial Normal Play Game

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This project investigated the winning values of the following Nim variant: There exists set $A$ and set $B$, with some number of elements in each. On a player's turn, the player may reduce set $A$ by $x$, set $B$ by $y$, or both sets by $z$, where $x, y$, and $z$ are positive integers. The player who takes the last turn, or reduces $A$ and $B$ to zero, wins. "Safe spots" are winning values of sets $A$ and $B$, such that the player who reduces the sets to the safe spot values will win. By analyzing the relationships between safe spots, a sequence was derived to represent the difference between adjacent safe spots. Subsequently, a recursive equation was developed to represent the sequence. The average distance between safe spots was calculated and found to be equal to phi, where phi is equal to $f \_n / f \_(n-1)$ as $n$ goes to infinity, where $f \_n$ is the $n t h$ Fibonacci number. Because of the correlation to the Fibonacci series, the project results may have applications in biology, number theory, geometry, and other fields of math and science.

