

Higher Jacobi Identities

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Any Lie algebra by definition satisfies the following identities $[x_1, x_2] + [x_2, x_1] = 0$ and $[x_1, x_2, x_3] + [x_2, x_3, x_1] + [x_3, x_1, x_2] = 0$, where $[a_1, \dots, a_n]$ denotes the left-normed Lie bracket. Moreover, it is easy to check that any Lie algebra satisfies one more identity: $[x_1, x_2, x_3, x_4] + [x_2, x_1, x_4, x_3] + [x_4, x_3, x_2, x_1] + [x_3, x_4, x_1, x_2] = 0$. This motivates the following definition. A subset T of the symmetric group S_n is said to be Jacoby if any Lie algebra satisfies the similar identity where the sum is taken over all permutations in T . Similarly, a subset T is said to be 2-Jacoby if the identity is satisfied in any Lie algebra over a field of characteristic 2. The work is devoted to an investigation of Jacoby and 2-Jacoby subsets. A big family of Jacoby subsets $T_{\{k, l, n\}}$ of S_n was constructed. Moreover, a 'vector space of all relations between left-normed brackets' was constructed and a basis of the space induced by the subsets $T_{\{k, 1, n\}}$ was found. It was proved that the class of 2-Jacoby subsets is closed under some operations. In particular, in contrast to the class of usual Jacoby subsets, the class of 2-Jacoby subsets is closed under symmetric difference, which makes it a $\mathbb{Z}/2$ -vector space. Moreover, it was proved that any 2-Jacoby subset can be obtained as a symmetric difference of several Jacoby subsets.