Conjecture of Maximum Number of Minimum-Area Triangles Determined by N Lattice Points in No-Three-in-Line Situation

Li, Xuanlin

The study of extremal problems in triangle areas, since A. Oppenheim in 1967, has been going on for almost five decades. In the project, a new general case in lattice grid is proposed: Given a set of n (n is greater than or equal to 3) lattice points in the plane and no any three points are allowed to be collinear, what is the maximum number of triangles that have the same smallest area among the C(n,3) triangles formed? Let f(n) be the greatest number of such triangles among n points. Two planes were set up, and each of the n lattice points in the original plane was projected to a unique line in the new plane. It was then proved that for any n, f(n) was between linear and quadratic. Then, a construction of n=2^k (k=2,3…8) lattice points was given to conjecture the non-linear bound of f(n). The upper bound was conjectured to be O(n ln n).