Geometric Interpretation of Hadamard Matrix Operator

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A Hadamard matrix is a square matrix that consists of 1 and -1 and whose rows are orthogonal. From the one hand, they are very useful (error-correcting codes: Reed–Muller codes, Walsh functions). From the other hand, the purely mathematical problem about possible sizes of Hadamard matrices still needs to be solved. The most important classes of Hadamard matrices are Sylvester matrices and Paley matrices. We investigate the operators defined by these two classes of matrices in terms of their geometry. We prove that the operator corresponding to a Sylvester matrix is the composition of an explicit Homothetic transformation and a symmetry relative to an explicit subspace of the Euclidean space. Moreover, we prove that the operator corresponding to a Paley matrix is the composition of an explicit Homothetic transformation and an explicit rotation of the Euclidean space.