Quivers of Some Classes of Group Algebras

Bogdanov, Ivan

VAGNER, MAKSIM

A quiver is a finite directed multigraph. However, this simple graph theory notion has an important algebraic interpretation. Each finite-dimensional algebra over an algebraically closed field has a quiver associated with it. We can give a definition of quiver in algebraic language: for each finite-dimensional algebra A over an algebraically closed field k there exists a unique algebra path quiver kQ/I, where the category of modules is equivalent to the category of modules A. Purpose of the research The quiver is the most important invariant of algebra. For example, algebra is semi-prime if and only if its quiver has no arrows. Also, homology dimension of algebra can not be greater than the maximum length of the path in the quiver. So, it is natural to study quivers of different types of algebras. One of the most interesting classes of finite-dimensional algebras is the class of group algebras of the finite group. Let us consider a group algebra over some field. If a field characteristic does not divide the group order then group algebra is semi-prime and its quiver has no arrows. But if the characteristic divides the group order, then the quiver is not trivial and highly dependent on the group. If one takes an Abelian p-group (where p is the characteristic of the field), then they can construct a group algebra for it. The purpose of our project is to develop a method of building quivers of group algebras. The problem of quiver building can be reduced to two problems – studying how to build a quiver of cyclic group algebras and developing a method of building quivers for tensor products of group algebras of finite groups. As a result, a method for building quivers of Abelian p-groups' algebras was developed, which is the main result of our project.