

The Game with Stones and "Generalized Fibonacci Sequence"

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I have researched an old Chinese game with stones. The rules of the game: There are two piles of stones, m stones in the first pile, and n in the second one. There are two players, who take the stones one by one. In one move, they can take any stones from the first heap or from the second or from the two heaps equally. The winner is the player who will take the latest stone. Our aim is to determine the winner for different sizes of piles, in other words, for numbers m and n . Let's examine the equivalent game on the board to the game with stones to gain the aim. It will help us to get the answer clearly and beautifully: There is a board, which consists of $m \times n$ squares and a figure in the lower right corner of it. Two players move this figure one by one to any number of squares in one of three directions: left, up, diagonally left-up. The winner is the player, who is the first to put the figure in the upper left corner. In the game on the board all losing positions for the player are the squares, which are located along two straight lines, which angular coefficient is connected with Fibonacci numbers. It turned out, that mathematics have already been investigating the game and main results have been discovered, but I proposed a new geometric method, which allowed to prove the main results by unified approach. I have described all winning and losing positions and proved three theorems: 1. There is exactly one losing square in each row, column and diagonal of the board. 2. The display of winning and losing positions on the bisector of the left-upper corner of the Board forms the "generalized Fibonacci sequence". 3. My method allowed to get formula for coordinates of losing positions. I have enjoyed this work and I would like to continue this research.