Tortoise or Hare? Improving Accuracy of Frequency, Amplitude, and Phase with Explicit Integration Rather than Fast Fourier Transforms

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Fourier transform methods are used to analyze functions and data sets to provide frequencies, amplitudes, and phases of underlying oscillatory components. Fast Fourier Transform (FFT) methods offer speed advantages over evaluation of explicit integrals (EI) that define Fourier transforms. This project compares frequency, amplitude, and phase accuracy of the two methods over a wide array of data sets including cosine series with and without random noise and a variety of physical data sets, including atmospheric CO2 concentrations, tides, temperatures, sound waveforms, and atomic spectra. The FFT uses MIT's FFTW3 library. Explicit integration uses code from Numerical Recipes in C. Results support the hypothesis that EI methods are more accurate than FFT methods. Root mean squared errors range from 5 to 10 times higher when determining frequency by FFT, 1.7 to 60 times higher for amplitude, and 6 to 10 times higher for phase. The ability to compute more accurate Fourier transforms has promise for improved data analysis in many fields, including more sensitive assessment of hypotheses in the environmental sciences related to CO2 concentrations and temperature. Other methods are available to address different weaknesses in FFTs; however, the EI method always produces the most accurate output possible for a given data set.