What Number Cannot Be Realized as the Number of Regions Divided by n Straight Lines?

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Given n straight lines in the plane, Roberts in 1889 gave a formula to count the number of regions formed by an arbitrary arrangement of the n lines. While the minimum and the maximum number of Roberts' formula can be easily determined and they form the two endpoints of an interval J, not every natural number in J can be realized as the number of regions divided by n lines. In other words, one will notice that there are "gaps" in the interval J. We prove that, when at least half of the lines, that is n/2, n/2+1, n/2+2 lines etc., are required to parallel each other or meet at the same point, the rest of the lines can be accordingly arranged so that the number of regions divided by the lines will form a closed subinterval in J. By taking the union of all the resulting subintervals, its complement will be the only locations where the gaps may possibly occur. Then by relaxing the condition so that fewer lines parallel each other or intersect at a point, we have derived a lower bound on the number of regions that can be used to prove the existence of some gap subintervals into which the plane cannot be divided. This result extends the partial results of Ivanov in 2010 and that of Arnold in 2013 on this problem. It is the best estimation so far for the location of the gaps where Roberts' formula cannot take.

Awards Won:

American Mathematical Society: First Award of $1,000