

# A Method to Calculate Exact Values of n th Power of the Irrational Number Phi

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This project created an effective mathematical method that simplifies the power elevating process for the golden ratio,  $(\Phi)$ . When using irrational numbers, approximations are used in utilizing a finite quantity of these digits. High exponentiation make results less exact, adding a long and tedious process. Assuming  $(\phi)$  is a solution to  $[x^2-x-1=0]$ , then, in combination with the replacement property and multiplicative equality, you will find a relationship to the values of the powers of the number  $(\Phi)$ . It started with the equation  $[(\Phi)^2-(\Phi)-1=0]$ , cleared  $[(\Phi)^2]$  and the result was,  $[(\Phi)^2=(\Phi) +1]$ , then  $(\Phi)$  is multiplied by the complete equation  $[(\Phi)*((\Phi)^2=(\Phi)+1)]$ , resulting in,  $[(\Phi)^3=(\Phi)^2+(\Phi)]$ ,  $[(\Phi)^2]$  was replaced by  $[(\Phi)+1]$ ,  $[(\Phi)^3=(\Phi)+1+(\Phi)]$ , sum the similar terms and resulted in  $[(\Phi)^3=2(\Phi)+1]$ . The procedure was repeated, and the relationship of the exponent with their coefficients and constants was noted in the form of  $[(\Phi)^n=a*(\Phi)+b]$ , where  $n$  represented the power of  $(\Phi)$ , The numbers  $a$  and  $b$  represent constants. It was found that can be used the simple algorithm of the Fibonacci Sequence to find the powers of the irrational number  $(\Phi)$ . A computer algorithm was later created based in the new calculation of powers of  $(\Phi)$ . In addition, it was found that the idea can be generalized to any irrational solution of a quadratic equation, which would help in the teaching of Algebra and Number Theory.