The Base Dependent Behavior of Kaprekar's Routine: A Theoretical and Computational Study Revealing New Regularities

Hanover, Daniel

Consider the following process: Take any four-digit number which has at least two distinct digits. Then, rearrange the digits of the original number in ascending and descending order, take these two numbers, and find the difference between the two. Finally, repeat this routine using the difference as the new four-digit number. In 1949, D. R. Kaprekar became the first to discover that this process, known as the Kaprekar Routine, would always yield 6174 within 7 iterations. Most research done on the behavior of this process was conducted in the latter half of the 20th century, in which computing power available to researchers was minimal. In this study, the author developed a series of C++ programs to analyze the paths integers followed to their respective Kaprekar's Constant. Surprisingly, it was determined from this program that the most commonly required number of iterations required to reach Kaprekar's Constant for 3-digit integers was consistently 3, regardless of base. When loaded as a matrix, the iteration requirement data demonstrates a precise recurring relationship reminiscent of Pascal's Triangle.