

Characterizing the Constructible N-Division Points of the Rational C-Hypocycloids through Straightedge and Compass Constructions

Mani, Nitya

One of the most famous mathematical problems concerns constructing figures with the aid of an unmarked straightedge and compass, specifically constructing regular polygons. Such construction problems concerning regular n -gons can be generalized to consider the n -division points of any closed curve C (n points dividing C into pieces of equal arc length). This year, I worked towards three major goals in the field of n -division point constructions: understanding the constructible n -division points of hypocycloids, characterizing the constructible n -division points of an entire family of curves for the first time in this field, and resolving a longstanding unaddressed disparity concerning the presence (or lack thereof) of a closed curve C when considering the constructible n -division points of C . I used Galois theory, abstract algebra, and algebraic number theory in order to characterize constructible numbers and prove two theorems. I showed that the n -division points of all rational c -hypocycloids are constructible with an unmarked straightedge and compass for all integers n , given a pre-drawn hypocycloid. I also considered the question of constructibility of n -division points of hypocycloids without a pre-drawn hypocycloid in the case of a tricuspoid, concluding that the n -division points of a tricuspoid are only constructible in this manner when $n \mid 6$ (the n -division points are always constructible when given a drawn tricuspoid).

Awards Won:

American Mathematical Society: First Award of \$600

European Organization for Nuclear Research-CERN: Third Award \$500