Functional Neural Networks Evaluated by Weierstrass Polynomials

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Artificial neural networks (ANNs) proposed by McCulloch and Pitts are machine learning algorithms that are able to approximate any mapping or function when given a dataset. ANNs model the human brain's capacity to store relationships between raw input and abstract concepts through dendritic connections, and in the early days of machine learning were very popular. The most pervasively used algorithm, the feed-forward ANN, is extremely extensible and applicable in many scientific fields, but cannot handle a large majority of datasets with continuous values. Hence, we explore a theoretical construction in which the number of neurons approaches infinity and call this new algorithm the functional neural network (FNN). We show the set of all ANNs to be a subset of all FNNs, and then subsequently prove that FNNs can approximate any function. Out of this proof, we provide novel insight into how the dendrite weights of ANNs form, an unsolved problem in machine learning which led to ANNs fall from popularity in the research community during 90s. We then show FNNs to be approximators of operators K: C(X) -> C(X), completing the construction by making a continuous analogue to the error backpropagation algorithm for ANNs. We conclude the project by developing a fast FNN algorithm and implementing it in C++ for open source application in scientific application. We use our API on multiple continuous and discrete datasets to demonstrate the approximation of mathematical operations. In conclusion the development of FNNs as a novel algorithm for machine learning could both mean the resurgence of ANNs and open a whole new field of inquiry into the theory thereof.