

Preserving Algebraic Structures on Exact Infinity: Categories with the K-theory Functor

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The purpose of this paper is to establish a new perspective on the K-theory of exact infinity-categories. I show that if the definition of K-theory is slightly modified, one can interpret the K-theory of an exact infinity-category as a stable infinity-category, and not as a spectrum. Since spectra are stable infinity-categories with a single object, this new perspective strictly generalizes the classical viewpoint of K-theory. My formalism encompasses all the information about the K-theory of ring spectra into a single statement: $K: \text{Sp} \rightarrow \text{Sp}$. As an example of the relative simplicity of calculations in our formalism, in this paper, I compute the K-theory of an infinity-operad of modules, and show that it must be equivalent to an infinity-operad of modules itself! This calculation provides a sufficient condition for the conclusion of a question asked by Carlsson to hold. In addition, I use this computation to state a generic property of the K-theory of the sphere spectrum (which is an open problem). I conclude this paper by proving a derived counterpart of the Morita context in stable model categories (hitherto unknown), which can be used to compare different exact infinity-categories via their K-theories, since it is an invariant.