Categorizing Point Sets with No Empty Pentagons

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Motivated by a question proposed in combinatorial geometry, we attempt to categorize finite point sets that do not have a 5-hole (empty pentagon). We look at a finite set of points, P, and consider the case that these points are in strictly convex position. We use the Erdös-Szekeres Theorem in tandem with a construction of convex layers and the visibility graph of the point set to extract information regarding the existence of an empty pentagon. A second approach is taken using the Djoković-Winkler relation to check if the flip graph of the point set is isometric to a hypercube, thereby verifying if the point set has an empty pentagon. We attempt to construct sets such that the Djoković-Winkler relation is an equivalence relation, creating point sets with 5-holes. We then propose an extension of the Erdös-Szekeres Theorem into higher dimensions.