Signatures of Multiplicity Spaces in Tensor Products of sl2 and Uq(sl2) Representations

Kishore, Shashwat

Representation theory is a field of mathematics that studies objects in abstract algebra by representing them as matrices. Objects can often be represented in many ways, and the most useful representations are those of unitary type. This project studies multiplicity space signatures in tensor products of sl2 and quantum sl2 representations, along with their applications to the unitary representation theory of quantized quiver varieties and the topology of master functions. Three main results are obtained. The first main result is a complete classification of definite multiplicity spaces for generic tensor products of sl2 Verma modules. This provides a classification of a family of unitary representations of a basic quantized quiver variety, the first such classification for any quantized quiver variety. The second main result is the first real critical point lower bound for generic sl2 master functions. This bound gives a simple and asymptotically correct approximation for the number of real critical points of a generic sl2 master function. The third main result is a formula for multiplicity space signatures in tensor products of irreducible finite dimensional quantum sl2 representations. This formula also gives multiplicity space signatures in generic tensor products of sl2 Verma modules and generic tensor products of real quantum sl2 Verma modules. These advances combine methods from linear algebra, representation theory, and statistical mechanics, and have direct applications to knot theory along with connections to quantum physics and geometric representation theory.

Awards Won:
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