Continued Fractions of Quadratic Numbers

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Continued fractions have a long history behind them - their origin may be placed to the age of Euclid's algorithm for the greatest common divisor or even earlier. However, they experience a revival nowadays thanks to their applications in high-speed and high-accuracy computer arithmetic. The continued fraction is usually defined as the expression $a_0+b_0/(a_1+b_2/(a_2+b_3/(a_3+...)))$, where a_0 is an integer, $b_i=1$, and other variables are natural numbers. Every real number x can be written in the form of continued fraction; x's fraction is often simplified as $x=[a_1,a_2,a_3,...]$. According to Lagrange's theorem, an irrational number has a periodic continued fraction if and only if the number is a root of a quadratic equation with integer coefficients (we call such a root quadratic irrational). The goal of my project was to describe continued fractions of $N^{1/2}$, where N is a natural number. I defined N as $N=n^2+j$ where j is from the set $\{1,2,...,2n\}$, and using a help of Wolfram Mathematica I created a table of period lengths of continued fractions of particular $N^{1/2}$-s. I studied the data and tried to observe any laws which I could prove. I described all continued fractions of $N^{1/2}$ with period lengths 1, 2, and 3 and many fractions with different period lengths. I also discovered a connection between continued fractions and Fibonacci numbers and made several not-yet-proved hypotheses.

Awards Won:

Second Award of $2,000