## Continued Fractions of Quadratic Numbers

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Continued fractions have a long history behind them - their origin may be placed to the age of Euclid's algorithm for the greatest common divisor or even earlier. However, they experience a revival nowadays thanks to their applications in high-speed and high-accuracy computer arithmetic. The continued fraction is usually defined as the expression a_0+b_0/(a_1+b_2/(a_2+b_3/(a_3+...))), where $a \_0$ is an integer, $b \_i=1$, and other variables are natural numbers. Every real number $x$ can be written in the form of continued fraction; $x$ 's fraction is often simplified as $x=\left[a \_1, a \_2, a \_3, \ldots\right]$. According to Lagrange's theorem, an irrational number has a periodic continued fraction if and only if the number is a root of a quadratic equation with integer coefficients (we call such a root quadratic irrational). The goal of my project was to describe continued fractions of $N^{\wedge} 1 / 2$, where $N$ is a natural number. I defined $N$ as $N=n^{\wedge} 2+j$ where $j$ is from the set $\{1,2, \ldots, 2 n\}$, and using a help of Wolfram Mathematica I created a table of period lengths of continued fractions of particular $\mathrm{N}^{\wedge} 1 / 2-\mathrm{s}$. Istudied the data and tried to observe any laws which I could prove. I described all continued fractions of $\mathrm{N}^{\wedge} 1 / 2$ with period lengths 1,2 , and 3 and many fractions with different period lengths. I also discovered a connection between continued fractions and Fibonacci numbers and made several not-yet-proved hypotheses.

## Awards Won:

Second Award of \$2,000

