

# Weighted Catalan Numbers and Their Divisibility Properties

Shader, Sarah (School: DaVinci Academy of the Science and the Arts)

The weighted Catalan numbers, like the Catalan numbers, enumerate various mathematical objects. For example, the number of Morse links with  $n$  critical points is the  $n$ -th weighted Catalan number,  $L_n$ , with weights  $1^2, 3^2, 5^2, \dots, (2k+1)^2, \dots$ . This paper examines the conjecture made by Postnikov which involves examining the divisibility of  $L_n$  by powers of 3. This project gives an upper bound of  $2 \cdot 3^{(2r-7)}$  on the period of  $L_n$  modulo  $3^r$ , which supports Postnikov's conjecture that this period is  $2 \cdot 3^{(r-3)}$ . The results are proven by representing  $L_n$  using combinatorial structures called Dyck paths. Dyck paths of length  $n$  are broken into pieces using a process called partial flat path decomposition. This classifies paths according to the location of the steps corresponding to the weights divisible by  $3^2$  or the weight 1. Properties of partial flat paths are proven and this knowledge combined with the use of mathematical tools, specifically generating functions, lead to the main result.

## Awards Won:

American Mathematical Society: Second Award of \$1000