

Approaching Factorial Numbers by Refining Stirling's Formula

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This research focuses on refining the formula of James Stirling that gives an approximation of the actual value of a factorial (Stirling, 1730). A wide interval was selected for the value of n and evaluated in the Stirling formula to study its approximations and their errors. It was found that the Stirling formula is not forthcoming enough when the value of the selected n increase. This leads to investigate how to get a better approximation by manipulating the original formula, it was found that the addition of a factor provides a better approximation of computing the factorial number, this was given by exploring factors that satisfied $1 < f < 2$. The findings are validated with a mathematical program. The resulting formula is $\sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{(5977/500)n}\right)$, which offered much more accurate values. These values were placed in a table to clearly compare the respective approaches of both formulas and other two factorial approximation formulas by Burnside and Gosper, where it can be observed a remarkable improvement in the factorial approximation. Precision percentages were then calculated, Stirling's formula has 99.443182 % of accuracy and the improved formula offers 99.999334 % of accuracy. The accuracy of this formula has many applications in quantum statistics, which are used in marketing or economics. In conclusion, using data provided by a mathematical program it can be induce that Stirling's formula was refined by the addition of the factor.