On the Lower Central Series of PI-algebras

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This project is inspired by a recent breakthrough in the study of the lower central series filtration of unitary associative Lie algebras. We consider the successive quotients of elements (and ideals of elements) of the lower central series, as they encode important information about the level of non-commutativity of the starting algebra. Using the representation theory of the general linear group, we provide a comprehensive description of the lower central series properties of a previously unstudied large class of PI-algebras (algebras with polynomial identities), which consists of the relatively free algebras in the variety of algebras module a product of two commutators. Firstly, we prove the Equivalence Theorem and the Isomorphism Property which establish PI-algebras as unique in the study of lower central series. Secondly, we prove the Structure Theorem for the free metabelian associative algebra on n generators, which is the simplest (but vital for PI theory) case. Thirdly, we extend the Structure Theorem to the general class of PI-algebras, and describe their GL-module structure for two and three generators. This provides us with an algorithm that finds explicit bases for the vector spaces of quotients of the lower central series. Finally, we give specific examples of application of the novel approach. As the action of the general linear group naturally descends on the quotients we consider, the comprehensive description itself motivates the study of the lower central series of PI-algebras in the context of non-commutative algebra and representation theory. Judging by our computations with the mathematical software MAGMA, we believe that this research's approach may be applied to even larger classes of PI-algebras.

Awards Won:

Second Award of $2,000