Deriving Some Trigonometric Identities and Inequalities in Triangles Using Vieta's Theorem

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Most of the trigonometric identities and inequalities in triangles have complicated proofs, which are usually done by tedious trigonometric transformations. In this project, we focused on proving these identities and inequalities in simpler ways by applying algebra to geometry. First, we provide an identity connecting both the radius of the circumscribed circle, the radius of the inscribed circle, the semi-perimeter of a triangle and one of the interior angles of the triangle. This relationship is satisfied for each interior angle of the triangle; therefore, one can expect that this identity can be transformed into an algebraic equation of a third degree. Then, applying Vieta's Theorem on the roots of polynomial equations to these third degree equations, we obtain various trigonometric identities and inequalities. Our method confirms D'Alambert's Statement: Algebra is generous; she often gives more than is asked of her. Our method can be successfully applied in solving Olympiad problems.