On Two Letter Identities in Lie Rings

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Abstract algebra is a branch of mathematics that studies algebraic structures. Lie algebras theory is one of the most important and developing topic of abstract algebra. A Lie ring is a Lie algebra over integer numbers. These structures occur in different areas of science such as quantum physics, combinatorial group theory and geometry. In this work, we study the question of describing two letter identities of the form [A,a]=[B,b], where A and B are some expressions of a and b. For example, [[[a,b],b],a]=[[[a,b],a],b]]. We consider A and B as elements of a free Lie ring on "a" and "b" and assume that two identities are different if the corresponding pairs (A, B) are different. For example, we are not interested in the identity [[a,a],a]=[[b,b],b] because it is the same as [0,a]=[0,b]. Identities of this kind arise in algebraic topology and because of that their generalization is demanded. We described all possible identities with two "a" letters and any number of "b" letters. One of the main results of our work is an exact formula for a nontrivial identity with three "a" letters and arbitrary number of "b" letters that is divisible by three. The main result is a method of finding all possible identities with three "a" letters up to multiplication by a coefficient. This algorithm relies on combinatorial-geometric approach for this problem. We also proved that there are no nontrivial identities with odd length of commutators lesser than 9 and present the very first identity with odd number of elements in commutators . Moreover, we obtained some statements about ranks of a free abelian groups that generate such identities. These results create a connection between some combinatorial questions and Lie rings theory, providing a straight simplification in some cases.

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