

New Explicit Solution to the N-Queens Problem and the Millennium Problem

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The N-Queens Problem is the problem of placing N chess queens on a NxN chessboard so that no two queens attack each other. Calculation of the number of different solutions is related to the problem of completion the NxN chessboard with $m < N$ queens to complete board with N queens. All known algorithms stop working for $N \geq 1000$. In August 2017 a group of Scottish mathematicians proved that N-Queens Completion is NP-complete and suggested one million dollars for a polynomial-time algorithm or just more efficient algorithm than existing algorithms which solve this problem. The main results of my work are: 1) Composition of arrangements A and B is a queens arrangement obtained by insertion of B into queens positions of A. I find the criterion of being a composition of solutions a solution. Sufficiency was proved 100 years ago, and I prove the most difficult part - the necessity with new consequences. 2) Queen function with width k is a special map which is defined on partition of a segment $[1, N]$ by the k segments as an almost usual linear map. I prove that for any $N > 3$ there exists a solution which can be represented as Queen function with width fewer or equal to 3. And this estimate of width cannot be reduced. 3) Based on obtained results I formulate a hypothesis: if for $N-1$ and N there are no solutions which are compositions of smaller boards then there exists a set of solutions which generate all solutions by symmetry which can be represented as a Queen function with width fewer or equal to 3. If this hypothesis is true then for such N I can construct a polynomial-time algorithm which solves N-Queens Completion. This hypothesis raises a new problem: is the number of such N is infinite? In particular, is the number of primes of the form $2^k 3^l - 1$ is infinite?

Awards Won:

American Mathematical Society: Certificate of Honorable Mention