

# The Frequency and Distribution of Consecutive Quadratic Residues Modulo $p$

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First, the number of pairs of consecutive quadratic residues modulo an odd prime  $p$  are counted, using sums of the Legendre symbol. We then extend this to triples of consecutive quadratic residues modulo an odd prime  $p$ . We find explicit formulas for the number of such triples based on the value of  $p \pmod{8}$ , using Jacobsthal's theorem. We then show the existence of infinitely long sequences of consecutive quadratic residues for all sufficiently large primes  $p$ . We examine the sum of the Legendre symbol of  $n(n+1)(n+2)(n+3)$  as it ranges across all of  $\mathbb{Z}/p\mathbb{Z}$  using the Riemann-Hurwitz theorem and the Hasse-Weil bound, and make some conjectures as well. We extend this to even-degree polynomials and squarefree quartics.