Extension of Soddy's Hexlet:Number of Spheres Generated by Nested Hexlets

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We successfully extended Soddy's theorem for the original Soddy's hexlet to derive the number of spheres of what we refer to as the n-th generation hexlet: 6^n. The number of spheres composing a Soddy's hexlet is always six. This simple and interesting fact has motivated us to study the n-th generation hexlet, specifically, how the number of spheres changes as we repeatedly generate new spheres in the way suggested by Soddy's hexlet. We define the n-th generation hexlet inductively as an extension of Soddy's hexlet. For n = 1, the spheres are a first generation hexlet, corresponding to the original Soddy's hexlet. For n >= 2, we extract all pairs of spheres that can be used to form new core spheres from the spheres belonging to the hexlets of up to the (n-1)-th generation. For all of the extracted pairs, we generate spheres that are in contact with the new core pairs and the circumsphere, and let the set of newly generated spheres form the n-th generation hexlet. Using computer analysis, we concluded that this set contains 6^n spheres for n <= 5. We further verified this conclusion by using the inversion method and demonstrated that there are only two types of spheres belonging to the n-th generation hexlet with respect to the contact relationship of the spheres. Simplifying a recurrence formula on the numbers of spheres of the two types, we successfully proved that the number of spheres belonging to the n-th generation hexlet is 6^n.

Awards Won:

American Mathematical Society: First Award of $2,000