

# Asymptotics of Character Sums

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In this project, we aim to prove certain properties about a particular function  $c(n) = b_{nr}(n)$ . This is where  $b_n$  is a Boolean function with  $b_n$  being 1 if  $n = x^2 + y^2$  for some integers  $x$  and  $y$  or 0 otherwise and  $r_{\chi}(n)$  is the sum of all of the Dirichlet characters which are divisible by  $n$ . Since  $\chi$  will be clear from context, we will suppress the subscript throughout this manuscript. The function  $c(n)$  sums the all of the  $\chi$  values of the divisors of a certain number  $n$  if and only if  $n$  can be expressed as the sum of two squares. Therefore, the question we ask is the following: What are the asymptotics of the character sums of the function  $c(n)$ ? In order to investigate this problem, we first represent the character sum of  $r(n)$  as an asymptotic and prove that the asymptotic is roughly  $L(1, \chi)$  with a small error term. Additionally, we compute a representation for the character sum  $c(n)$  as an Euler product, and also find error bounds on the asymptotic for the character sum.

## Awards Won:

American Mathematical Society: Second Award of \$1,000

Fourth Award of \$500