The Hofstadter Butterfly on a Ring Lattice: Exploring the Discrete Case & Translating to a Continuous Model with a Bichromatic Sinusoidal Potential

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The Hofstadter Butterfly is a distinct fractal pattern in quantum physics that describes the energy eigenvalues of electrons subject to a magnetic field in a two-dimensional (2D) lattice. As part of this project, an alternative way to create the Hofstadter butterfly spectrum was studied with a ring-shaped 1D discrete lattice which was translated to a continuum physical model involving a bichromatic sinusoidal potential. The project was theoretical in nature, where neutral ultracold atoms trapped in this ring-shaped potential were modeled and described with quantum mechanics, and specifically with the Schrödinger equation. The discrete lattice model was described by the relevant Hamiltonian matrix and the eigenvalues found by numerical diagonalization. The study examined how the variation of the different lattice parameters like inter-site coupling strength, onsite modulation potential, and lattice depth affected the spectrum in the continuum model, and found the optimal parameters. For the physically realistic continuum model, the study found that the coupling strength has an inverse relation to the lattice depth, which was computed through the overlap integral. Consequently, the optimal butterfly spectrum was found to occur for higher values of the lattice depth when the onsite modulation strength was proportionately decreased. This study demonstrated that the Hofstadter model can be implemented with realistic potentials on a 1D ring-shaped lattice, and also determined the necessary optimal spectral conditions. The results of this project can provide a simpler yet effective way to fully implement this seminal model in physics that has been known for 40 years in theory.