Monodromy Groups of Indecomposable Rational Functions

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The most important geometric invariant of a degree-n complex rational function f(X) is its monodromy group, which is a set of permutations of n objects. This monodromy group determines several properties of f(X). A fundamental problem is to classify all degree-n rational functions which have special behavior, meaning that their monodromy group G is not one of the two typical groups, namely A_n or S_n. Many mathematicians have studied this problem, including Oscar Zariski, John Thompson, Robert Guralnick, and Michael Aschbacher. In this research, I bring this problem near completion by solving it when G is in any of the classes of groups which previously seemed intractable. I introduce new techniques combining methods from algebraic geometry, Galois theory, group theory, representation theory, and combinatorics. The classification of rational functions with special behavior will have many consequences, including far-reaching generalizations of Mazur's theorem on uniform boundedness of rational torsion on elliptic curves and Nevanlinna's theorem on uniqueness of meromorphic functions with prescribed preimages of five points. This improved understanding of rational functions has potential significance in various fields of science and engineering where rational functions arise. My findings have a wide range of potential applications, from creating faster, more secure algorithms for telecommunications to designing safer infrastructures, like bridges resistant to strong winds. My work is broadly relevant to a variety of problems in cryptography and various mechanical systems.

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