Testing Chebyshev's Bias for Prime Numbers Up to 5*10^15

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Presented research belongs to number theory and is a part of a more ambitious project to test Chebyshev's bias numerically for prime numbers up to 1.84*10^19. Chebyshev's bias, known from 1853, is closely related to the prime number counting function, Generalized Riemann Hypothesis and zeros of Dirichlet L-function – the foundation of the modern number theory. Between 1957 and 1996 it was numerically tested up to 10^12 for all small moduli and corresponding residues (referred as "prime number races"). For several moduli and residues there were no any zones found where Chebyshev's bias would be ever violated. In the beginning of 2000s several theoretical models predicting such zones were developed. At that time, most predicted zones were located beyond the reach of available technologies. The main objective of this research was to expand 5000 times the previously tested 10^12 range for 15 "most biased prime number races" established theoretically in 2013. Several computer programs using fast prime number generator called primesieve were developed and tested to implement the project. In total 25 new sign-changing zones for 9 prime number races were found between 10^12 and 5*10^15 that compared with only 11 zones for 4 races found between 1957 and 1996. Of all 25 discovered zones, 4 zones are the first found for their corresponding races and present the most scientific interest. Each found zone supports the validity of Generalized Riemann Hypothesis. Some of the discovered zones, especially small and narrow, happened to be much lower than was predicted by theoretical models. Therefore, these models need revision. At the same time, the theoretical models predicted large and wide zones well. All results are contributed to OEIS as 18 separate sequences.