

Mersenne Primes: An Exploratory Study of Patterns and Some New Conjectures

Lohan, Rajat (School: Delhi Public School)

Introduction Mersenne Primes:- A number of the form $2^n - 1$ where n is a natural number is a Mersenne number. If its value is prime then it is known as a Mersenne Prime. Mersenne primes are named after the mathematician Marin Mersenne who discovered these around 1550 BC with the help of Perfect Numbers. Some of the initial Mersenne primes were – $2^2-1=3$; $2^3-1=7$; $2^5-1=31$; $2^7-1=127$; $2^{13}-1=8191$ and $2^{17}-1=131071$ The Largest known Mersenne Prime Number is $2^{(82,589,933)}-1$ and it contains 24,862,048 digits. It took approximately 12 months of calculations to find this Prime Number.

*Methodology 1. In $2^n - 1 = X$, I have observed some patterns between the power of 2, that is n , and the number of digits in its value, that is, X . The patterns were: - $2^{31} \sim 32 + 2 \times 31 = 94$ {i.e. 2^{501} has 94 number of digits} $2^{32} \sim 33 + 2 \times 32 = 97$ {i.e. 2^{511} has 97 number of digits} $2^{33} \sim 34 + 2 \times 33 = 100$ {i.e. 2^{521} has 100 number of digits} 2. I have studied about the Euclid's proof of prime numbers which leads me to propose an easy method to find prime numbers. 3. I have formed a quadratic equation whose roots are the two prime numbers used in the RSA encryption system. *Results and Findings 1. I have formed patterns and two relations between the values of 2^n , $2^n - 1$ and their number of digits. They help me to eliminate 33.33% numbers which will never be a prime number. The relations are: - For Power: $An = 1 + (n-1)10$ For number of digits: - $An = 1 + (n-1)3$ 2. I have proposed a new and easy method of predicting big prime numbers. 3. I am able to find the values for the two primes in the RSA encryption system non computationally. I am working on its computational method.