# Analysis of the Error Convergence and Efficiency of Numerical Quadrature Algorithms for Approximating Different Integrals

Brosula, Raphael (School: Strawberry Crest High School)

Many functions do not have elementary antiderivatives and different analytical methods to evaluate the definite integral fail. These functions appear in fields such as biology, physics and engineering. To determine the integral, numerical quadrature algorithms were used to approximate the integral via a weighted sum of the product of a weight function and evaluation at nodes along the domain of integration. Different algorithms have different error convergences and runtimes for different functions. The project uses a robust set of functions to analyze the error convergence and efficiency of different numerical quadrature algorithms such as Gaussian and Clenshaw-Curtis through different MATLAB implementations of the algorithms. Functions were split into two categories based on their error convergence: half-precision (accuracy to four decimal digits) and double-precision (accuracy to sixteen decimal digits). To measure accuracy, there was a comparison between the least number of evaluation points needed to reach half-precision or double precision. To measure runtime, the average runtime of each algorithm up to 500 evaluation points determined runtime trends. Theoretically, Gaussian quadratures converge faster than Clenshaw-Curtis by a factor-of-two; however, experimental results show that for many different functions the Gaussian and Clenshaw-Curtis quadrature algorithms converge similarly, contrary to mathematical prediction. Clenshaw-Curtis quadrature however, operated in linear computational time whereas Gaussian operates in polynomial computational time, indicating that Clenshaw-Curtis is an optimal candidate due to its runtime efficiency. Further studies can be made by evaluating more quadrature algorithms in comparison.