Generating Set for Nonzero Determinant Links under Skein Relation

Karan, Aayush

Traditionally introduced in terms of advanced topological constructions, many link invariants may also be defined in much simpler terms given their values on a few initial links and a recursive formula on a skein triangle. Then the crucial question to ask is how many initial values are necessary to completely determine such a link invariant. We focus on a specific class of invariants known as nonzero determinant link invariants, defined only for links which do not evaluate to zero on the link determinant. We restate our objective by considering a set $S$ of links subject to the condition that if any three nonzero determinant links belong to a skein triangle, any two of these belonging to $S$ implies that the third also belongs to $S$. Then we aim to determine a minimal set of initial generators so that $S$ is the set of all links with nonzero determinant. We show that only the unknot is required as a generator if the skein triangle is unoriented. For oriented skein triangles, we show that the unknot and Hopf link orientations form a set of generators. These results highly improve the efficiency of computing nonzero determinant link invariants and thus improve the feasibility of distinguishing between link structures. This completely resolves a question posed by David Mullins that has remained open for 30 years. Outside of mathematics, my results have intriguing applications in DNA-enzyme action and potential applications in quantum field theory.

Awards Won:
United Technologies Corporation: Each winning project will receive $3,000 in shares of UTC common stock.
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