# Generating Set for Nonzero Determinant Links under Skein Relation

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Traditionally introduced in terms of advanced topological constructions, many link invariants may also be defined in much simpler terms given their values on a few initial links and a recursive formula on a skein triangle. Then the crucial question to ask is how many initial values are necessary to completely determine such a link invariant. We focus on a specific class of invariants known as nonzero determinant link invariants, defined only for links which do not evaluate to zero on the link determinant. We restate our objective by considering a set S of links subject to the condition that if any three nonzero determinant links belong to a skein triangle, any two of these belonging to S implies that the third also belongs to S. Then we aim to determine a minimal set of initial generators so that S is the set of all links with nonzero determinant. We show that only the unknot is required as a generator if the skein triangle is unoriented. For oriented skein triangles, we show that the unknot and Hopf link orientations form a set of generators. These results highly improve the efficiency of computing nonzero determinant link invariants and thus improve the feasibility of distinguishing between link structures. This completely resolves a question posed by David Mullins that has remained open for 30 years. Outside of mathematics, my results have intriguing applications in DNA-enzyme action and potential applications in quantum field theory.

**Awards Won:**
Second Award of $1,500
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