

# Finding Chebyshev-Type Functions

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This project is motivated by studying Chebyshev polynomials such as  $\cos(f_0^m(mx))$  is a polynomial of  $\cos(f_0^m(x))$ . It is interesting to know if there are other non-constant continuous functions  $f$  having the similar property, i.e.,  $f(mx)$  is a polynomial of  $f(x)$ . In this study, we are able to characterize all non-constant continuous functions  $f$  from the positive real number set to the complex number set such that for all positive integer  $n$ , there exist polynomial  $P(x)$  satisfying  $f(mx)=P(f(x))$ . Furthermore, we generalize the problem by replacing the positive integer set with an arbitrary subset of the positive integer set.

## Awards Won:

American Mathematical Society: Second Award of \$1,000

Third Award of \$1,000