Loop Spaces, P-Curvature, and Homotopy

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Given a Vector Bundle (Coherent Sheaf) with Integrable Connection on a variety in positive characteristic, its p-curvature measures the amount of horizontal sections of the bundle and its vanishing at reductions at maximal ideals (corresponding to reductions at primes) is conjectured to be equivalent to the connection having a full set of solutions in characteristic 0. In his study of this conjecture for generic curves, Ananth Shankar (2016) showed that images of simple closed loops in the monodromy representation defined by a connection are finite assuming the triviality condition on reduced p-curvatures. In this work, we consider families of free loop spaces defined on covers of algebraic varieties with connections to attempt to understand this relationship in higher generality. Using the Leibnitz product rule for integrable connections, we construct a binary operation on free loop spaces defined by a given connection, and show that it gives an Abelian group structure in certain cases, and also show a correspondence between torsion in this group and singular points of the connection. We then consider this operation taken in the reduced setting, and give an application of the Frobenius Theorem on pullbacks of connections. Lastly, we also consider relationships between free loop spaces and stratified bundles, which play the part of bundles with flat connections in an equicharacteristic setting, and consider homotopy theoretic aspects of the category of free loop spaces associated to covers of a given base variety as an application of Fabien Morel’s Homotopy Theory of Schemes.

Awards Won:
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