Searching for Hidden Black Holes: An Investigation of Chaotic Regimes in Non-Linearly Coupled Harmonic Oscillators

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Chaos is ubiquitous in nature but hard to study in full generality, making case studies crucial. It is also important in holography, a presumed component of quantum gravity, where it can identify black holes in higher dimensions. This study investigated the characteristics and determinants of chaos in a classical system with D = 1 spacetime dimensions, and whether it could be interpreted as a model of gravity in D = 2 spacetime dimensions. Non-linear differential equations were derived through Lagrangian mechanics, and chaos quantified using Lyapunov exponents. Algorithms for analysis of chaotic dynamics were constructed in Mathematica, and mathematical analyses performed. Notably, chaotic regimes were characterized by more distinct states in phase space, caused by full energy transfer between oscillators. Chaos increased with the coupling parameter, total energy, and potential energy, while being unaffected by kinetic energy. Transitions back into stable regimes were found for larger total energy, but not for stronger coupling. Analytical results showed that the system cannot be chaotic for identical or decoupled oscillators, and that regime transitions can be precisely predicted. This study identified core characteristics of chaos, and suggested that full energy transfer might be a fundamental hallmark of chaos in a coupled system. In addition, determinants and non-determinants were identified. Further research will investigate the analytical formula relating chaos in this system to a black hole on a line, as based on the work of Maldacena et al. (2016).

Awards Won:

Third Award of $1,000