

# On a Generalization of Artin's Conjecture for Primitive Roots in Gaussian Integers

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We propose a generalization of Artin's conjecture on primitive roots to the ring  $\mathbb{Z}[i]$  of Gaussian integers. We conjecture that for a fixed positive integer  $q$ , every non-zero Gaussian integer  $a$  that is not  $+1$  or  $-1$ , generates a cyclic subgroup of the multiplicative group of  $\mathbb{Z}[i]/\mathfrak{p}$  of residue index  $q$  for infinitely many prime ideals  $\mathfrak{p}$ . In several special cases we reduce it either to the classical Artin's conjecture, or to its extension for near-primitive roots, the Golomb's conjecture. We divide the conjecture into three cases: when  $a$  is on the real axis, when  $a$  is on the imaginary axis, and when  $a$  is on neither axes. We conclude by showing that for every  $a$ , we have that the sum of  $d(a,q)$  over all positive  $q$  equals 1, where  $d(a,q)$  is density of the prime ideals  $\mathfrak{p}$  yielding subgroups of index precisely  $q$ .