An Application of Group Theory to Number Theory

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We consider regular n-gons, free to float in 2D space. This allows them a rotational property. Now, we consider the problem of calculating the number of distinct a-colorings of its vertices. The objective of the project is to find an explicit formula for the number of distinct a-colorings of regular n-gons free to move in 2D space, and use the fact that such a number must be an integer to derive some number theoretic results. In the project, we develop an abstraction of the geometric idea of transformations using modular arithmetic, which is the main innovation. We label the vertices of the n-gon in a cyclic manner and find a way to mathematically represent the transformations. Then, we use a well-known result in group-theory to develop an explicit computational formula and the result is used to prove a generalization of two theorems in number theory. I believe that such an explicit formula and generalizations of the theorems has not appeared in print before. Two of the books referenced in the creation of this project have chapters on the topic the project is on, but they do not feature such an explicit formula. Hence, this project is a way of developing something really significant from a seemingly trivial problem.