Amazing Triangle

Chkhatarashvili, Lizi (School: Georgian-American High School)

Sharoiani, Luka (School: Georgian-American High School)

Vashakmadze, David (School: Georgian-American High School)

In this project we set out to prove that in isosceles triangles The medians, heights and bisectors are equal. The two line segments – formed from connecting the bases of medians, heights and bisectors with opposite sides or with its continuation – are equal. We discussed the inversed question: if two line segments formed by connecting the bases of triangle’s bisectors are equal, is the original triangle isosceles or not? We investigated three different cases for medians, heights and bisectors. Given that if the circle which passes through the bases of medians intersects with the base of triangle, then the triangle is isosceles. We then proved that if the circle passes through the bases of bisectors and intersects with one of the sides of the triangle, this triangle does not necessarily have to be isosceles, and we discovered that all non-isosceles triangles for which this is also true. For this, we discussed some cases. For the given triangle, we used the bisector and tangent properties and we wrote the equations: (e1) $\left(\frac{ac}{a+b}\right)^{2} = \frac{ac}{b+c}\left(\frac{ac}{b+c}+x\right)$ (e2) $\left(\frac{bc}{a+b}\right)^{2} = \frac{bc}{a+c}\left(\frac{bc}{a+c}+y\right)$ After simplifying the equations we obtain that a = b and that the triangle is isosceles. After considering the following: $\tilde{x}=a+b$ and $\tilde{y}=ab$. With the help of Viet’s Theorem, we get that a and b must be roots of the following equation: $t^2-\tilde{x}t+(\tilde{x}^3+\tilde{x}^2-\tilde{x}-1)=0$. we got that the critical points are: $\tilde{x}_1 = (-3-\sqrt{57})/12 \approx -0,879$ minimum $\tilde{x}_2 = (-3+\sqrt{57})/12 \approx 0,379$ maximum. We proved that if the circle goes through the bases of bisectors, intersects one of the sides and the triangle is non-isosceles, then the angle is obtuse and is within $138°35'25'' < C < 139°17'13''$.