Unit Groups, Quotients, and New Perspectives on tau-l-factorizations

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In 2011, Anderson and Frazier introduced the theory of generalized factorizations in integral domains, which is based on studying restrictions of the multiplicative operation of an integral domain. A special case of these constructions involves factorizations over the integers, where non-zero non-unit elements are allowed to be multiplied if pairwise their difference is divisible by a given positive integer n. Using number theory and group theory, Hernandez-Espiet (2019) characterized a structure of irreducible elements in such factorizations. This work comes as the latest in a sequence of different forms of approach to the problem, and is based on an algorithm using the structure of the group of units of the integers modulo the principal ideal generated by a positive integer n and of a homomorphic image of this group. In this work, we begin by generalizing the construction of these groups for arbitrary ideals in integral domains, and later study their structure in the context of classical constructions in algebra such as Character Theory and Galois theory. Our results highlight the connections between the groups of units mentioned before, Cyclotomic Fields, and the proof of Quadratic Reciprocity.