

On a Variation of the Witsenhausen Problem Concerning Maximal $\pi/2$ -Avoiding Spherical Sets

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The problem of finding the largest $\pi/2$ -avoiding spherical set first appeared in the AMS monthly journal in 1974. H. S. Witsenhausen asked readers to determine $\alpha(n) = (|U|)/(|S^{n-1}|)$, where U is the largest $\pi/2$ -avoiding set in the n dimensional sphere S^{n-1} , that is, the largest set on the sphere in n dimensions containing no orthogonal vectors. We look at a variation of this problem: determine $\alpha(n,k) = (|U|)/(|S^{n-1}|)$, where U is the largest set on the n dimensional sphere S^{n-1} that contains no k mutually orthogonal vectors. We begin by specializing the problem to the case $n=k=3$ by considering spherical sets that avoid three mutually orthogonal vectors in 3D in an attempt to determine $\alpha(3,3)$ which allows for more interesting configurations and visualization. We find the largest spherical subset U not containing three mutually orthogonal vectors in the following cases: (i) U is a large double cap; (ii) U is a centered band; or (iii) U is a wedge shape. We then prove a generalized lower bound for $\alpha(n,k)$ for any $n \geq 3$. By taking U , the union of two double caps and a centered band, we increase the area further. The measures of these sets give a provable lower bound for $\alpha(3,3)$. Finally, by smoothing the band using computational results, we find a configuration created by the intersection of elliptical cylinders with the sphere that gives an even better lower bound for $\alpha(3,3)$. We conjecture this to be the largest spherical set for $\alpha(3,3)$, and one that can be generalized to higher dimensions.