

# Diophantus Equations and Partially Ordered Sets

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In Alzer, Horst and Luca's paper it is shown that the Diophantine equation  $(k!)^n + k^n = (n!)^k + n^k$  only has the trivial solution  $n=k$ , and  $(k!)^n - k^n = (n!)^k - n^k$  only has the solutions  $n=k$ ,  $(n, k) = (1, 2)$ , and  $(2, 1)$ . In this article we find all solutions of the Diophantine Equations  $a_1!a_2! \cdots a_n! \pm a_1a_2 \cdots a_n = b_1!b_2! \cdots b_k! \pm b_1b_2 \cdots b_k$ , where  $a_i$  majorizes  $b_i$ . Furthermore we find a sufficient condition on a function  $f: \mathbb{N} \rightarrow \mathbb{R}^+$  to guarantee that  $f$  gives a monotone function on the POSET of all finite sequences of natural numbers. We then use that to solve other Diophantine equations involving factorials and generalize the results of Sándor's paper. We also explore similar Diophantine Equations for the Fibonacci Sequence and other sequences of natural numbers given by linear recursions of the form  $A_{(n+2)} = aA_{(n+1)} + bA_n$ .

## Awards Won:

Third Award of \$1,000