Cracking the Infinite Shuffle: Solving the Kimberling Sequence Problem

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The Kimberling shuffle is an integer sequence generated by the shuffling and expulsion of numbers. From the sequence of natural numbers, 1 is expelled and the first integer in front (2) is placed first and the first behind (nothing so it is skipped) is placed second, this gives the sequence 2,3,4,5… which will be the base for the next stage. Here the second element is expelled, and the numbers behind and in front will be shuffled so the third stage would be 4,2,5,6,7… and so on. The expelled elements (1,3,5…) form the Kimberling sequence. An open problem is whether every positive integer will eventually appear in this sequence, one that has remained unsolved for nearly 30 years. In this project, the nature of the sequence and problem is investigated using computational methods (measuring times and patterns required to reach certain values) and varied analysis, and from different angles: through that of limiting inequalities, number-theoretic properties, and algebraic manipulation. Interesting results linked to this problem were obtained, namely, a method through which every rational number might be represented as the result of repeated applications of two distinct operations on a fraction and a unique Collatz-styled simplification of any number using its divisibility and parity. Finally, the proof for the Kimberling sequence problem is also discussed in relation to these different methods. This investigation not only helps to solve this age-old problem but also has implications in number theory and in the investigations of a myriad of other recursive infinite sequences.

Awards Won:

Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: Second Award of $1,000