

# Solving Some of the Algebraic Equations Using Trigonometry

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It is known that attempts to solve some equations and systems of equations by standard methods often cause great difficulties. However, it is more convenient to solve these equations and system of equations using the properties of trigonometric functions, and it is much easier to solve the problem. In this project, I have brought the solution of algebraic examples to the solution of trigonometric examples using trigonometric substitutions. These substitutions depend on the given equation, the system of equations, or the algebraic expression that needs to be simplified. For instance, if from the given conditions it is clear that the possible set of values of  $x$  variable is obtained by the  $|x| \leq 1$  inequality, substitutions such as  $x = \sin a$ ,  $a, \epsilon [\pi/2; \pi/2]$  or  $x = \cos a$ ,  $a, \epsilon [0; \pi]$  are more convenient to apply than traditional methods. In this case, whether which one to use depends on the given equation. If the variable can obtain any values, substitutions like  $x = \tan a$ ,  $a, \epsilon (-\pi/2; \pi/2)$  and  $x = \operatorname{ctg} a$ ,  $a, \epsilon (0; \pi)$  can be applied. For example, 1.  $\sqrt{1-x^2} = 2x^2 - 1 + 2x\sqrt{1-x^2}$  solve the given equation. 2. In the range of  $[0; 1]$   $8x(1-2x^2)(8x^4 - 8x^2 + 1) = 1$  how many roots does this equation have? 3.  $\begin{cases} 2x + x^2 y = y^2 z \\ y^2 z + z^2 x = x^2 y \end{cases}$  determine all the real triads  $(x; y; z)$  of the given system. These types of equations are solved using trigonometry. Since it is very difficult to solve these equations analytically, I think that the solution I have shown can be very useful for people who are deeply interested in mathematics.