Curve Optimization Using Curvature Based Models with Calculus of Variations

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This research approached the optimization of train curves in a mathematical way by using differential geometry and the calculus of variations. Our objective was to find the travel time-minimizing smooth curve which connects two parallel straight railroads. We constructed the arc line model – a point-symmetric curve composed of straight lines and circular arcs – and showed that the travel time is minimized when only two circular arcs are used. In order to minimize the time integral, we conjectured and proved the symmetry theorem: an odd function minimizes the length and travel time between two parallel train curves. The ideas of calculus of variations, odd-even decomposition, polynomial approximation, and uniform convergence were applied to obtain a necessary condition. The extreme value theorem was applied to the space of Fourier coefficients of even functions to show the existence of a global minimum. The minimizing property of odd functions can be generalized to arc length integrals with positive real exponents for parametrized curves. In order to consider real-world situations, we developed multiple speed formulas that depend on the curvature at a point on the train curve. Calculus of variations was applied to obtain the fourth-order nonlinear differential equation that the optimal train curve must satisfy. We solved the differential equations by using Mathematica and MATLAB's boundary value problem solver bvp4c. The optimal curves obtained from BVP and polynomial approximation were applied for the modeling of real-world train curves and showed enhancements in travel time.

Awards Won:
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