Classification of Small Orthomodular Posets in Quantum Logic through Clique Structures of Dacey Graphs

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Dacey established a class of simple, finite graphs, called Dacey graphs, which maps bijectively to the set of finite orthomodular posets, posets where the join of orthogonal elements always exists and satisfy constructive properties. Orthomodular posets are important in the study of quantum logics and quantum experimentation. The class of orthomodular posets, in Von Neumann's formulation of quantum mechanics, uniquely define finite experiments or in quantum mechanics under the framework of empirical logic. The discovery and enumeration of Dacey graphs, subsequently new quantum mechanical experimental apparatuses, could potentially be used to disprove hidden variable theories. A graph is Dacey if the maximal cliques intersect coherently under Dacey's mapping. We begin an enumeration of Dacey graphs by presenting methods of partitioning Dacey graphs into equivalence classes based on the structures of maximal clique intersections and characterizing Dacey graphs which are edge-covered by m maximal cliques. This leads to a complete classification of Dacey graphs for $m \leq 4$ and examples of small orthomodular posets not commonly found in literature. We then explore the implications of applying the conditions of Dacey graphs as local conditions to show each Dacey graph is "locally" Dacey and investigate the bounds of such a local condition. By identifying graph theoretical constraints and properties of Dacey graphs, we can extend the results to better understanding the nature of orthomodular posets.

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