

The Structure of the Positive Monoid of Integer-Valued Polynomials Evaluated at an Algebraic Number

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Motivated by the characterization of symmetric monoidal functors between Deligne categories, the additive monoid $R_+(x)$ is defined as the set of all nonnegative integer linear combinations of binomial coefficients $\binom{x}{n}$ for non-negative integers n . My project was concerned with the inquiry into the structure of $R_+(\alpha)$ for complex numbers α . I proved that this object is a ring if and only if α is an algebraic number that is not a nonnegative integer. I also gave two explicit representations of $R_+(\alpha)$, for both algebraic integers and general algebraic numbers α . One is in terms of inequalities for the valuations with respect to certain prime ideals and the other is in terms of explicitly constructed generators α_p . In particular, all algebraic integers generated by α are also contained in this ring. I also obtained a particularly good simplification of the structure of $R_+(\alpha)$ that works for all primes not dividing the discriminant of its minimal polynomial. This implies that a complete computationally effective characterization is obtained for every α , for all but finitely many prime numbers. Moreover, it leads to a particularly simple description of $R_+(\alpha)$ for both quadratic algebraic numbers and roots of unity α . I also recovered a result from Harman and Kalinov about the transitivity of $R_+(\alpha)$, only using combinatorial techniques, whereas their proof uses category theory.

Awards Won:

Fourth Award of \$500

American Mathematical Society: Third Award of \$500