Analytical Solutions of Compartmental Epidemic Models and Their Application to Parameter Estimation

Goz, Marton (School: Synge Street CBS)
Harhangi, Yaduvir (School: Synge Street CBS)

The SIR model is one of the simplest examples of a class of mathematical models, called 'compartmental models', that are used to predict the spread of an epidemic. A number of efficient numerical methods are available for solving sets of simultaneous differential equations of the type that arise in compartmental models. In this project an approximate analytical solution of the set of three simultaneous differential equations comprising an SIR model has been derived. The solution is 'approximate' because its validity depends on a certain condition being satisfied, namely, the fraction of the population in the removed class must be very small. In the case of the early evolution of the Covid-19 epidemic in Ireland and other countries this condition is easily met. Accurate estimates of the parameters of an SIR model predicting the spread of Covid-19 in the Republic of Ireland were obtained by comparing the daily reports of the observed cumulative number of detected Covid-19 infections published by the Irish Health Protection Surveillance Centre with the values predicted by the approximate analytical solutions. New approximate analytical solutions of other compartmental models have been derived. A number of new compartmental models are considered. Some of these new models address an important question: is it possible to find a way to allow an acceptable rate of transmission of Covid-19 in the younger, 'active' population, thereby limiting the damaging effects of lockdowns on the economy of a country, and at the same time provide a safe environment for the 'vulnerable' (cocooning) group?