Study on the Solution Set of Knot Colorings

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This study presents the coloring of knot and link diagrams that satisfies certain conditions. For p-colorability, each crossing leads to a linear congruence: therefore a diagram with n crossings satisfies n congruences, creating a matrix. We first devised an algorithm that presents general solutions from DT notations, and used it for knots with 12 crossings and under. Analyzing general solution sets shows that p-coloring solutions appear as linear combinations of free variables, and leads to the relationship between number of free variables and modular p rank, and number of solutions. To extend colorability to composites, we defined ‘Coloring matrix’ and used it to study the solution set itself. We defined ‘essentiality’ as follows. When all the solutions are divided by the number t which is lager than 1, we call the solution inessential. When the solution is not inessential, we call it essential. We then analyzed sets using essentiality and cardinality, and presented and proved determinant-coloring solution formula. The results includes how to derive the coloring properties and solution sets of any natural number from certain results. Essentiality and cardinality are invariants, which can be proved using the transformation rules of knot diagram matrices made by Reidemeister moves. We also extended p-coloring and m-coloring for links. We created the solution algorithm for links, and found that knot coloring properties are also valid for links.

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